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Weakly warped extra dimensions and SN1987A

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Abstract: The neutrino pulse from SN1987A provides one of the most rigourous constraints on models of extra dimensions. Previously, calculations have been done to bound the size of these extra dimensions in the case when the metric was factorizable. Here we consider the case of 2 'weakly warped' extra dimensions. We find that even though weak warping seems only to affect the zero mode this can have a measurable effect on the supernovae bounds. In any braneworld model such warping is necessarily present and as such should be taken into account in supernovae bounds and in searches for corrections to newtonian gravity.

Keywords: Extra Large Dimensions, Related Astrophysics (Neutron Stars, Supernovae etc.).

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Contents

1. Introduction

Recently there has been an explosion of interest in theories with extra spacetime dimensions as possible solutions to the gauge hierarchy problem and the cosmological constant problem, see for example [1]–[4]. These models can be tested in various ways depending on the size and warping of the extra dimensions. Such tests include, observing corrections to a $1/r$ gravitational potential at short distances, observing missing p_T or new spin 2 resonances in high energy colliders or measuring the shape of the neutrino pulse from supernovae.

For the case of 2 extra dimensions, perhaps the most rigourous constraint comes from the neutrino pulse observed from SN1987A. Various analyses have been carried out for the case of toroidally compactified extra dimensions [5]–[7]. However, if the Standard Model fields are constrained to lie on a 4 dimensional brane in the higher dimensional spacetime then the energy density situated on the brane will cause a warping in the higher dimensions. The compactification will no longer be toroidal but instead will be some curved, compact manifold. Therefore, we consider a weakly warped compactification to investigate how the warping affects the bounds coming from the supernovae data.

In order to do this we consider a model similar to that of Chen, Luty and Ponton [10]. This model has two extra dimensions, which is the most interesting phenomenologically. We are interested in the effects of the light KK modes of the graviton on the neutrino pulse of SN1987A and we ignore the radion since it has no KK modes [8]. The supernova bounds are sensitive to all the low lying KK modes (those whose energy is approximately less than the temperature in the core of the star). As such the effect of any one particular mode coupling with gravitational strength is negligible.

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The strength of a KK mode's coupling to matter is determined by the ratio of its wavefunction on the 3-brane to that of the zero mode. All the KK modes whose energy is greater than the warping scale will be unaffected by the warping. Thus, one might expect very little effect, for weak warping, since very few modes are directly affected. However, introducing a warping introduces a potential for the zero mode graviton [4], giving its wavefunction a profile in the extra dimensions. This results in the normalization for the zero mode changing.

The normalization of the zero mode sets the scale of gravitational interactions, $G_N = N_0^2$, which in turn affects the strength of the KK couplings. A KK mode's coupling to matter on our brane is determined by the mode's value at our brane. The zero mode's coupling to our brane determines the strength of gravity. So the ratio of a KK mode's value at the origin to that of the zero mode determines the mode's coupling to matter relative to the strength of gravity. Thus, if the KK modes are unaffected by warping and N_0 becomes smaller then the KK modes actually now couple with greater than gravitational strength. The effect of weak warping can be modeled by a tower of KK modes whose masses don't change from the unwarped case but whose couplings are all shifted.

We find that, for the model considered here, the zero mode's coupling is decreased by warping and so the effect of the KK modes increases.[1] This results in a change in the supernova bounds between the (unnatural) unwarped case and the warped case. Notice that the effect of the warping is to change the coupling strength of the KK modes from that expected for the unwarped case. In particular this means that in searches for short distance corrections to gravity one should not expect the coupling of the lightest KK mode, α, to be simply determined by the number or geometry of the extra dimensions, [12, 13].

This paper is organized as follows. In section 2 we describe the model and solve the equations of motion for the KK modes of the graviton. In section 3 we describe how the KK modes affect the neutrino pulse from SN1987A. In section 4 we discuss the case of stronger warping and we conclude in section 5.

2. The model

We consider a variation of the (6 dimensional) 'spaceneedle metric' of Chacko and Nelson [9]. The inner 4-brane on which the standard model fields live is moved into the origin, and becomes a 3-brane, and we impose a Z_2 symmetry around the outer 4-brane so that the radial extra dimension ends again on an identical copy of the original 3-brane. Such a scenario has also been considered by Chen, Luty and

[1] It is also conceivable to have models where the warping increases the zero mode coupling, for instance in models where we live at a maximum of the warp factor rather than a minimum.

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Ponton [10]. Explicitly we consider a metric of the form

$$ds^2 = f(r)\eta_{\mu\nu}dx^\mu dx^\nu + s(r)d\theta^2 + dr^2\,, \tag{2.1}$$

where μ, ν run over our 4 spacetime dimensions (we take a 'mostly plus metric') and θ, r span the 2 extra dimensions.

The stress energy on the 3-brane is given by[2] $T_{ab} = \mathrm{diag}(T_4, T_4, T_4, T_4, 0, 0)$ and on the 4-brane by $T_{ab} = \mathrm{diag}(T_5, T_5, T_5, T_5, T_{5,\theta}, 0)$. This model has a global deficit angle that is proportional to the 3-brane tension and there is one overall fine tuning that is necessary to make the 3-brane flat.

Solving Einstein's equations for this metric and matching across the boundaries at the branes produces a solution of the form,

$$f = \cosh^{4/5}\left(\frac{5}{2}\sqrt{k}r\right)\,, \qquad s = \frac{f'^2}{f}\,, \tag{2.2}$$

where $k = -\Lambda_6/10M_6^4$ with Λ_6 being the bulk cosmological constant which we will take to be negative and M_6 the fundamental Planck scale.

We are interested in the properties for the graviton KK modes and as such will need to solve the equations of motion for a perturbation about this solution. Consider a perturbation of the form,

$$ds^2 = (f(r)\eta_{\mu\nu} + h_{\mu\nu}(r, x^\alpha))dx^\mu dx^\nu + s(r)d\theta^2 + dr^2\,. \tag{2.3}$$

We expand $h_{\mu\nu}(r, x^\alpha)$ in plane waves,

$$h_{\mu\nu}(r, x^\alpha) = \sum_p e^{ip.x} h_p(r)\,, \tag{2.4}$$

and using Einstein's equations we find that h (where we henceforth drop the subscript p for convenience) obeys the equation of motion,

$$-h'' - \frac{1}{2}\frac{s'}{s}h' - \left(\frac{f'}{f}\right)^2 h + 10kh = \frac{m^2}{f}h\,, \tag{2.5}$$

where $p^2 = \eta_{\alpha\beta}p^\alpha p^\beta = -m^2$. We are ignoring modes that have θ dependence since they couple to momentum in the θ direction and thus they do not couple to particles living on the 3-brane. Such modes can be pair produced by SM particles on the brane but these processes will be suppressed. The boundary conditions imposed on h by the presence of the branes at $r = 0$ and $r = L$ respectively are,

$$h'(0) = 0 \qquad \text{and} \qquad \Delta\frac{h'}{h} = -\frac{T_{5,\theta}}{2}\,. \tag{2.6}$$

[2] Following notation of Chen, Luty and Ponton.

As it stands (2.5) is difficult to solve so we make the simplifying assumption that the warping is weak, i.e. $\sqrt{k}L < 1$. We will only keep the dominant terms in an expansion in $\sqrt{k}L$. Upon solving this simplified equation of motion we will see that the solution is entirely consistent with the assumptions we have made. For weak warping (2.5) becomes,

$$-h'' - \frac{1}{r}h' + 10kh = m^2 h\,. \tag{2.7}$$

The solution, after imposing the boundary condition at the 3-brane, is

$$h = N_m J_0\left(\sqrt{m^2 - 10k}r\right)\,. \tag{2.8}$$

Where the allowed values of m^2 will be specified by applying the boundary conditions at the 4-brane. For weak warping this is,

$$\left.\frac{dJ_0/dr}{J_0}\right|_{r=L} = 5kL - \frac{125}{12}k^2L^3 + O(k^3L^5)\,. \tag{2.9}$$

This results in the masses being given by,

$$m_n^2 = \left(\frac{\nu_{1n}}{L}\right)^2 + \frac{125}{6}k^2L^2 + O(k^3L^4)\,. \tag{2.10}$$

Where ν_{1n} is the nth zero of J_1; for large masses the spacing of these zeros becomes π.

In order to find N we must normalize the wave functions for the KK modes i.e.,

$$\begin{aligned} N_n^{-2} &= \int_0^{\theta_{\max}} d\theta \int_0^L rdr\left|J_0(\nu+\delta)(\frac{r}{L}))\right|^2 \\ &= \theta_{\max}L^2 \int_0^1 dzz\,|J_0((\nu+\delta)z)|^2 \\ &= \frac{\theta_{\max}L^2}{(1+\delta')^2}\int_0^{1+\delta'} dww\,|J_0(\nu w)|^2\ ,\ \text{where } \delta' = \delta/\nu\,, \end{aligned} \tag{2.11}$$

where $\delta = -5kL^2/\nu + O(k^2L^4)$. Now, $\int_0^{1+\delta'} dwwJ_0^2 = F(\delta')$ where we can then expand $F(\delta')$ in a Taylor series about $\delta' = 0$. We find that,

$$N_n^{-2} = L^2\theta_{\max}J_0^2(\nu_{1n})\left\{\frac{1}{2} + 2\delta'^2 + O(\delta'^3)\right\}\,. \tag{2.12}$$

We can carry out a similar calculation for the zero mode and we find that the normalization N_0 is given by,

$$N_0^2 = \frac{2}{\theta_{\max}L^2}\left[1 - \frac{5}{2}L^2k\right]\,. \tag{2.13}$$

From the above we see that, to leading order in kL^2, the only effect of the warping is on the normalization of the zero mode, the spacing and normalization of the higher modes are unaffected. Thus, as mentioned above, this now means that the KK modes couple with greater than gravitational strength since it is the KK mode to zero mode ratio that determines the strength of coupling. The density of states of the KK modes is unchanged.

Intuitively, when we introduce warping the quantum mechanics problem that corresponds to the equation of motion of the graviton acquires a potential that is no longer flat but has a 'volcano potential' as in the RS models. Now since in our model the graviton zero mode is peaked around the other brane (an effect of the volcano potential) and the KK modes are orthogonal to the zero mode the KK modes will be stronger at our brane than in the case with no warping. Thus they interact more strongly. Conversely, if we had been living at a maximum of the warp factor the coupling of the KK modes would be decreased by warping. The effect of light warping can be modeled simply by altering the coupling strength of the tower of KK modes of the graviton, keeping their spacing the same as the unwarped case. This allows us to easily adapt results derived for flat compactifications. Let us calculate the effect on supernova bounds.

3. Bounds

A theory with compactified extra dimensions can be thought of in 2 different ways depending on the energy regime we are interested in. If only the first few KK modes can be excited then the theory can be thought of from a 4D point of view. We have the massless graviton and a tower of KK modes. Varying the warping and brane spacing shifts the spacing and coupling of the KK modes. If we are at a sufficiently high energy such that many of the KK modes can be excited it makes sense to think of the theory in its fundamental 6D description.

From the above we saw the effect of weak warping was to raise the coupling of the KK modes without shifting their masses. This means that in short distance gravity experiments where only the first few KK modes are accessible the strengths of these modes are not simply related to the geometry of the compactification [12, 13]. Instead the warping that is necessarily present in braneworld models changes the coupling of these modes from that expected due to geometry alone.

However, in a supernova the temperature is on the order of 30 MeV. For weak warping many KK modes are excited, so the theory is best viewed from a 6D point of view. The effects of the KK tower have to be summed up and result in a correction to newtonian gravity,

$$F_{\text{Newton}} \to \frac{m_1 m_2}{M_{pl}^2 r^2}\left(1 + \frac{M_{pl}^2}{M_*^4}\frac{1}{r^2}\right) . \tag{3.1}$$

The basic principle used to place supernova bounds on the sizes and warpings of the extra dimensions is the idea that having extra light modes to excite in supernovae explosions could change the shape of the neutrino pulse associated with these explosions. Since we know the shape of the neutrino pulse from SN1987A reasonably well any new physics that would significantly alter the pulse from that observed can be ruled out. The extra light modes present (with masses up to the average energy in the proto-neutron star) could be excited and thus would 'steal' energy from the neutrinos. A rule of thumb is that if the emissivity of these new energy loss processes is higher than $10^{19} ergs/g/s$ then the neutrino signal would be sufficiently different from that observed that we would be able to rule the processes out [11].

The dominant processes, contributing to this energy loss and involving the graviton, is bremsstrahlung of KK gravitons from nucleons, $NN \to NNh$. Hanhart *et al.* [5] computed these processes in a model independent way, relating their emissivities to measured nucleon-nucleon cross sections. However, in their calculation the extra dimensions were compactified on a flat torus. We can easily adapt their calculation to the case of weak warping. There are two changes that need to be made to their calculation,

1. We have warping in our extra dimensions [5] does not.
2. Hanhart *et al.* compactified the extra dimensions on a torus, whereas we have compactified the extra dimensions on a cone.[3]

First the effect of warping, this changes the coupling of the KK modes. In [5] the coupling is defined to be $\kappa_h = \sqrt{32\pi G_N}$ but for us it is $\kappa^2 = 32\pi G_N/(J_0^2(\nu_{1n}))(1 + 5/2kL^2) \equiv 32\pi G_N/(J_0^2(\nu_{1n}))g_{KK}^2$. Secondly, the change in geometry results in a change in the density of states of the KK modes,

$$\begin{aligned} L^2\Omega_1 \int dm m &\to \frac{L}{\pi} g_{KK}^2 \int \frac{dm}{J_0^2(mL)} \\ &= \frac{1}{2} L^2 g_{KK}^2 \int m dm \,, \end{aligned} \tag{3.2}$$

where in the last step we have used the asymptotic form of the Bessel function, $J_0^2(x) \sim 2/\pi x \cos^2(x - \pi/4)$ so $J_0^2(\nu_{1n}) \sim 2/\pi m L$. g_{KK}, the KK mode coupling, can be varied independently of the brane spacing and in the limit of zero warping it is 1.

Thus we can relate the flat results, $F(R)$, with a torus of radius R to the warped results, $W(k, L)$ with warping scale k and radius L:

$$W(k, L) = F(R) * (R^2\Omega_1)^{-1} g_{KK}^2 \frac{L^2}{2} \,, \tag{3.3}$$

where Ω_1 is the surface area of a unit 1-sphere.

[3] The extra dimensions are topologically equivalent to a sphere, they have a deficit angle singularity at the origin due to the 3 brane and a curvature singularity at the orbifold point due to the 4 brane. In the limit of zero warping the extra dimensions are a 'wedge' cut from a sphere.

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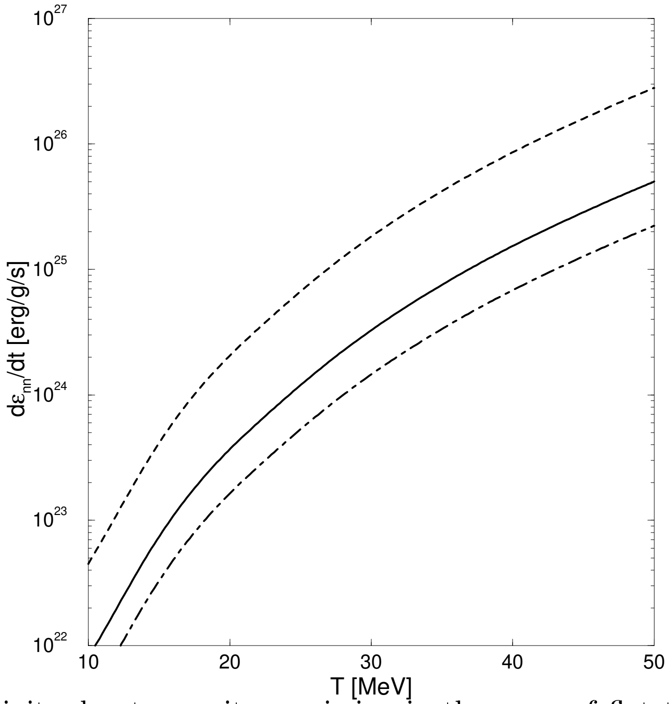

Figure 1: Emissivity due to graviton emission in the cases of flat toroidal compactification (dashed line), flat conical compactification (dot-dashed line) and warped conical compactification (solid line).

In order to see the effects of the warping and the different geometry on the results of ref. [5] we plot emissivity against temperature for various models. As in [5] we take the density of nucleons in the neutron star to be that of nuclear matter, $0.16 fm^{-3}$. Figure 1 shows emissivity for the various models. The dashed line shows the emissivity for the model of [5] which has flat toroidal extra dimensions, the dot-dashed line is for the conical geometry with no warping and the solid line is for the conical geometry with a warping of $k = 0.5\, mm^{-2}$, $R = L = 1\, mm$ in all cases.

Comparing the two similar geometries represented by the dot-dashed and the solid lines we can see the effect of warping is to increase the energy emission through graviton KK emission, as described earlier. In going from the flat toroidal compactification to the warped conical compactification there is also a change in geometry. This change in geometry overrides the effect of the warping and results in the energy loss being greater in the flat case. However, if two similar geometries are considered the effect is determined by warping alone and the energy loss increases.

Eq. (3.1) can be derived by summing up the effect of all KK mode exchange, so $M_*^{-4} \sim g_{KK}^2$. Together with (3.3) this tells us that the emissivity bound on extra dimensions is an immediate bound on M_*.

In a supernova the available energies are sufficiently high that it effectively 'sees' all of the KK modes and so experiences the true 6D theory. Thus, for weak warping, the supernova can place bounds on M_* beyond those already known from short distance gravity experiments. In a warped model both the coupling and the spacing are parameters, there are many combinations that result in the same M_*. For weak warping, with $k \approx 1/L^2$, the bounds on the brane spacing for various supernova temperatures are:

$$\begin{array}{lll} L \lesssim & 10^{-2}\, mm & T = 20\,\mathrm{MeV} \\ L \lesssim & 2\times 10^{-3}\, mm & T = 30\,\mathrm{MeV} \\ L \lesssim & 5\times 10^{-4}\, mm & T = 50\,\mathrm{MeV} \end{array} \tag{3.4}$$

4. Stronger warping

So far we have been interested in the case $\sqrt{k}L < 1$ but there are other regimes with stronger warping that can also be considered. The assumption $k \gg 1/L^2$ makes (2.5) more tractable. In this limit (2.5) becomes,

$$-h'' - \sqrt{k}h' + 6kh = 2^{4/5} m^2 e^{-\sqrt{k}r} h\,. \tag{4.1}$$

This has a solution,

$$h = Ae^{-1/2\sqrt{k}r}\left(BJ_{5/2}\left(\frac{2^{2/5}me^{-\sqrt{k}r}}{\sqrt{k}}\right) + Y_{5/2}\left(\frac{2^{2/5}me^{-\sqrt{k}r}}{\sqrt{k}}\right)\right)\,, \tag{4.2}$$

A and B are constants to be determined by boundary conditions.

Calculating the mass and couplings of the KK modes for this solution becomes a difficult problem. It is necessary to match at the boundaries and to normalize the wavefunction to find A, B and the allowed masses. We have not been able to find an analytic closed form for the emissivity in this regime. To do so would require a numerical investigation and as such is beyond the scope of this paper. For the low lying KK modes whose masses are small enough that the small argument limit of the Bessel functions can be used we find that the boundary condition at $r = L$ implies $B \approx 3\left(2^{2/5}me^{-\sqrt{k}L}/\sqrt{k}\right)$. Thus the coupling strength of the KK modes becomes energy dependent. The effects of warping are no longer determined by the zero mode alone.

If $k < MeV^2$ the majority of the KK modes excited in the supernova are unaffected by the warping. The emission of KK modes with $m \gg \sqrt{k}$ can be thought of as bremsstrahlung from nucleons into a 6 dimensional spacetime. In this fundamental 6D picture the gravitational coupling strength of the KK modes is determined by M_*. The energy loss to KK emission is determined by M_*. Although the supernova data could not be used to bound L or k separately, it can be used to place bounds on M_*. The bound on M_* is essentially unchanged from that of the flat case.

If $k \gg \mathrm{MeV}^2$ then all the KK modes that will be excited in the supernova will feel the warping. This scenario is more like that of ref. [3] and the supernova data is of little use in placing bounds in such a situation.

5. Conclusions

Warping is a necessary feature of braneworld scenarios and we have demonstrated that even very weak warping can have a measurable affect on low energy physics. For warping on the scale of the size of the extra dimensions, $k \sim 1/L^2$, we find that the leading order effect is to change the coupling of the KK modes. The mode spacing is relatively unchanged since their masses are $\gg L^{-1}$ so they are unaffected by the warping.

This change in KK mode coupling affects bounds coming from SN1987A and short distance gravity experiments. For the particular model considered here we found that this strengthens the bounds on L from those of the unwarped case. If we were living at a local maximum of the warp factor we would expect the bounds on L to be weakened. The latter case might be testable in short distance gravity experiments. In either case we expect the fundamental scale to be too high to be probed by accelerators.

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